10 SUSY Breaking and the Minimal Supersymmetric Standard Model

10.1 Tree Level Breaking

$$\langle 0|H|0\rangle > 0 \tag{10.1}$$

implies that supersymmetry is broken. So models where $F_i = 0$ and $D^a = 0$ cannot be simultaneously solved will have spontaneously broken SUSY.

The Fayet-Iliopoulos mechanism [3] uses a non-zero D-term for a $U(1)$ gauge group.

$$\mathcal{L}_{\rm FI} = \kappa^2 D \tag{10.2}$$

where κ is a constant parameter with dimensions of mass.

$$V = \frac{1}{2}D^2 - \kappa^2 D + gD\sum_i q_i \phi^{*i}\phi_i \tag{10.3}$$

$$D = \kappa^2 - g\sum_i q_i \phi^{*i}\phi_i. \tag{10.4}$$

If ϕ has large positive mass^2 terms, then $\langle\phi\rangle = 0$ and $D = \kappa^2$. In the MSSM however this would give vevs to squarks and sleptons

O'Raifeartaigh models [4] use non-zero F terms.

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2. \tag{10.5}$$

$$V = |F_1|^2 + |F_2|^2 + |F_3|^2; \tag{10.6}$$

$$F_1 = k - \frac{y}{2}\phi_3^{*2}; \qquad F_2 = -m\phi_3^*; \qquad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*. \tag{10.7}$$

The minimum of the potential is at $\phi_2 = \phi_3 = 0$ with ϕ_1 undetermined. $V = k^2$ at the minimum of the potential. Around $\phi_1 = 0$, the mass spectrum of scalars is

$$0,\ 0,\ m^2,\ m^2,\ m^2 - yk,\ m^2 + yk. \tag{10.8}$$

There are 3 fermions with masses

$$0,\ m,\ m. \tag{10.9}$$

Since SUSY is broken, quantum corrections will give a mass to the scalars. The effective potential for the scalars can be calculated a la Coleman-Weinberg [5]. However the massless fermion ψ_1 stays massless since it is the Nambu-Goldstone particle for the broken SUSY generator, the *goldstino*.

Fayet-Iliopoulos and O'Raifeartaigh models set the scale of SUSY breaking by a dimensionful parameter (κ or k) which is put in by hand. To get a SUSY breaking scale that is naturally small compared to M_{Pl} we need an asymptotically-free gauge theory that gets strong at some scale

$$\Lambda \sim e^{-8\pi^2/(bg_0^2)} M_{Pl} \tag{10.10}$$

and breaks SUSY non-perturbatively.

We also need new fields beyond the MSSM fields whose auxiliary fields get VEV's, since a D-term VEV for $U(1)_Y$ does not lead to an acceptable spectrum, and there is no gauge-singlet whose F-term could develop a VEV. The SUSY breaking field can't have renormalizable tree-level couplings to the MSSM fields. Supersymmetry does not allow (scalar)-(gaugino)-(gaugino) couplings. Also there is a sum rule for tree level breaking

$$\mathrm{Tr}[M^2_{\text{real scalars}}] = 2\mathrm{Tr}[M^2_{\text{chiral fermions}}]. \tag{10.11}$$

Thus we expect that SUSY breaking occurs in a "hidden sector" and is communicated by non-renormalizable interactions or through loops. If the interactions are flavor blind it is possible to suppress flavor changing neutral currents.

10.2 SUSY Breaking Scenarios

The two most popular scenarios for SUSY breaking are *gravity mediated* and *gauge mediated* SUSY breaking.

In the *gravity mediated* scenario, interactions with the SUSY breaking sector are suppressed by powers of M_{Pl}. If the hidden sector has a non-zero F component for some field,$\langle F \rangle$, then the soft terms in the visible sector should be roughly of order

$$m_{\text{soft}} \sim \frac{\langle F \rangle}{M_{Pl}}, \tag{10.12}$$

To get the weak scale we need $\sqrt{\langle F \rangle} \sim 10^{10}$ -10^{11} GeV. If SUSY is broken by a gaugino condensate $\langle 0|\lambda^a \lambda^b|0\rangle = \delta^{ab}\Lambda^3 \neq 0$. then

$$m_{\text{soft}} \sim \frac{\Lambda^3}{M_{Pl}^2}, \tag{10.13}$$

so $\Lambda \sim 10^{13}$ GeV.

In the gauge-mediated supersymmetry breaking scenario[8, 9],

$$m_{\text{soft}} \sim \frac{\alpha_a}{4\pi}\frac{\langle F\rangle}{M_{\text{mess}}} \tag{10.14}$$

where M_{mess} represents the masses of the messenger fields which couple to ordinary gauge interactions. If M_{mess} and $\sqrt{\langle F\rangle}$ are comparable, then the SUSY breaking scale can be as low as $\sqrt{\langle F\rangle} \sim 10^4$ -10^5 GeV

10.3 The Goldstino

Consider the fermions in a general model $\Psi = (\lambda^a, \psi_i)$. The mass matrix is

$$\mathbf{M}_{\text{fermion}} = \begin{pmatrix} 0 & \sqrt{2}g_a(\langle\phi^*\rangle T^a)^i \\ \sqrt{2}g_a(\langle\phi^*\rangle T^a)^j & \langle W^{ij}\rangle \end{pmatrix} \tag{10.15}$$

This matrix has a zero eigenvector

$$\widetilde{\Pi} = \begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle F_i\rangle \end{pmatrix}. \tag{10.16}$$

this can be shown using the facts that the superpotential is gauge invariant and

$$\langle \partial V/\partial\phi_i\rangle = 0 \tag{10.17}$$

The supercurrent conservation equation

$$0 = \partial_\mu J^\mu_\alpha = i\langle F\rangle(\sigma^\mu\partial_\mu\widetilde{\Pi}^\dagger)_\alpha + \partial_\mu j^\mu_\alpha + \ldots \tag{10.18}$$

implies

$$\mathcal{L}_{\text{goldstino}} = i\widetilde{\Pi}^\dagger\overline{\sigma}^\mu\partial_\mu\widetilde{\Pi} + \frac{1}{\langle F\rangle}(\widetilde{\Pi}\partial_\mu j^\mu + h.c.) \tag{10.19}$$

When one takes into account gravity, supersymmetry must be a local symmetry. This means that the spinor ϵ^α that parameterizes SUSY transformations is not a constant. This locally supersymmetric theory is called *supergravity* [6, 7]. It contains a spin-2 graviton and its spin-3/2 fermion superpartner called the gravitino, $\widetilde{\Psi}^\alpha_\mu$ which transforms inhomogeneously under local supersymmetry transformations:

$$\delta\widetilde{\Psi}^\alpha_\mu = -\partial_\mu\epsilon^\alpha + \ldots \tag{10.20}$$

The gravitino is like the "gauge" particle of local SUSY transformations, and when SUSY is spontaneously broken, the gravitino acquires a mass by "eating" the goldstino. This is the other *super-Higgs* mechanism. The gravitino mass is can be estimated as

$$m_{3/2} \sim \frac{\langle F \rangle}{M_{Pl}}, \tag{10.21}$$

In gravity-mediated SUSY breaking, the gravitino mass is comparable to m_{soft}. In gauge-mediated SUSY breaking the gravitino is much lighter than the MSSM sparticles if $M_{\text{mess}} \ll M_{Pl}$, so the gravitino is the LSP. The longitudinal components of the gravitino (the goldstino) have non-gravitational interactions. The decay rate of any sparticle $\widetilde{X}$ into its Standard Model partner X plus a goldstino $\widetilde{G}$ is given by

$$\Gamma(\widetilde{X} \to X\widetilde{G}) = \frac{m_{\widetilde{X}}^5}{16\pi\langle F \rangle^2}\left(1 - \frac{m_X^2}{m_{\widetilde{X}}^2}\right)^4. \tag{10.22}$$

If $m_{\widetilde{X}} \approx 100$ GeV, and $\sqrt{\langle F \rangle} < 10^6$ GeV [so $m_{3/2} < 1$ keV], then the decay $\widetilde{X} \to X\widetilde{G}$ can be observed in a collider.

10.4 Gravity-mediated SUSY Breaking

The effective soft-breaking Lagrangian below the Planck scale should be:

$$\begin{aligned}
\mathcal{L}_{\text{eff}} &= -\frac{1}{M_{Pl}} F_X \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + h.c. \\
&\quad -\frac{1}{M_{Pl}^2} F_X F_X^* \, k^i_j \phi_i \phi^{*j} \\
&\quad -\frac{1}{M_{Pl}} F_X \left(\frac{1}{6} y'^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'^{ij} \phi_i \phi_j\right) + h.c.
\end{aligned} \tag{10.23}$$

where F_X is from the hidden sector, and ϕ_i and λ^a are the scalar and gaugino fields in the visible sector.

It is usually assumed that there is a common $f_a = f$ for the three gauginos; that $k^i_j = k\delta^i_j$ is the same for all scalars; and that the other couplings are proportional to the corresponding superpotential parameters, so that $y'^{ijk} = \alpha y^{ijk}$ and $\mu'^{ij} = \beta \mu^{ij}$ with universal dimensionless constants α and β. Then one finds that the soft terms in can be written in terms of:

$$m_{1/2} = f\frac{\langle F_X \rangle}{M_{Pl}}; \qquad m_0^2 = k\frac{|\langle F_X \rangle|^2}{M_{Pl}^2}; \qquad A_0 = \alpha\frac{\langle F_X \rangle}{M_{Pl}}; \qquad B_0 = \beta\frac{\langle F_X \rangle}{M_{Pl}} \tag{10.24}$$

In terms of these, the soft SUSY breaking parameters in eq. (7.17) are:

$$M_3 = M_2 = M_1 = m_{1/2}; \tag{10.25}$$

$$\mathbf{m^2_Q} = \mathbf{m^2_{\bar{u}}} = \mathbf{m^2_{\bar{d}}} = \mathbf{m^2_L} = \mathbf{m^2_{\bar{e}}} = m_0^2\,\mathbf{1}; \quad m^2_{H_u} = m^2_{H_d} = m_0^2; \tag{10.26}$$

$$\mathbf{a_u} = A_0\mathbf{y_u}; \qquad \mathbf{a_d} = A_0\mathbf{y_d}; \qquad \mathbf{a_e} = A_0\mathbf{y_e}; \tag{10.27}$$

$$b = B_0\mu. \tag{10.28}$$

However equivalence principle (gravity is flavor blind) does not guarantee these universal terms.

Taking the four SUSY breaking parameters and μ and running them down from the unification scale (rather than the Planck scale as one would expect) is referred to as the *minimal supergravity* scenario.

References

[1] S.P. Martin, "A supersymmetry primer," hep-ph/9709356.

[2] H.E. Haber, "Introductory low-energy supersymmetry," hep-ph/9306207.

[3] P. Fayet and J. Iliopoulos, *Phys. Lett.* B **51**, 461 (1974); P. Fayet, *Nucl. Phys.* **B90**, 104 (1975).

[4] L. O'Raifeartaigh, *Nucl. Phys.* **B96**, 331 (1975).

[5] S. Coleman and E. Weinberg, *Phys. Rev.* D **7**, 1888 (1973).

[6] S. Ferrara, D.Z. Freedman and P. van Nieuwenhuizen, *Phys. Rev.* D **13**, 3214 (1976); S. Deser and B. Zumino, *Phys. Lett.* B **62**, 335 (1976); D.Z. Freedman and P. van Nieuwenhuizen, *Phys. Rev.* D **14**, 912 (1976); E. Cremmer et al., *Nucl. Phys.* **B147**, 105 (1979); J. Bagger, *Nucl. Phys.* **B211**, 302 (1983).

[7] E. Cremmer, S. Ferrara, L. Girardello, and A. van Proeyen, *Nucl. Phys.* **B212**, 413 (1983).

[8] M. Dine and W. Fischler, *Phys. Lett.* B **110**, 227 (1982); C.R. Nappi and B.A. Ovrut, *Phys. Lett.* B **113**, 175 (1982); L. Alvarez-Gaumé, M. Claudson, and M. B. Wise, *Nucl. Phys.* **B207**, 96 (1982).

[9] M. Dine, A. E. Nelson, *Phys. Rev.* D **48**, 1277 (1993); M. Dine, A.E. Nelson, Y. Shirman, *Phys. Rev.* D **51**, 1362 (1995); M. Dine, A.E. Nelson, Y. Nir, Y. Shirman, *Phys. Rev.* D **53**, 2658 (1996).